R&D Collaboration Networks in Mixed Oligopoly*

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Abstract

We develop a model of endogenous network formation in order to examine the incentives for R&D collaboration in a mixed oligopoly. Our analysis reveals that the complete network, where each firm collaborates with all others, is uniquely stable. When R&D subsidies are not available, in addition to the complete network, the private partial and the private-hub star networks are Pareto efficient. However, the complete network becomes the unique Pareto efficient network when R&D is subsidised. This result is in contrast with earlier contributions in private oligopoly where under strong market rivalry a conflict between stable and efficient networks is likely to occur. It also highlights the role of a public firm as policy instrument in aligning individual incentives for collaboration with the objective of efficiency, independently of whether R&D subsidies are provided by the regulator.

1 Introduction

In advanced industrial economies, R&D collaboration plays a crucial role for the creation, exploitation and diffusion of knowledge. Firms participating in collaboration agreements innovate more frequently than others and discover more original innovations (Beise and Stahl 1999). In doing so collaborating firms are able to increase their profitability and achieve superior economic performance than their non-collaborating counterparts.

An important feature of collaboration agreements is that they often engage both private and public firms. Mixed oligopoly is a very common form of market in Europe and in Japan following the introduction of competition into traditional state monopolies. An example of R&D collaboration in mixed oligopoly is the Norwegian industry for fuel cells and hydrogen technologies. In particular, Norway's portfolio includes a variety of R&D projects aimed at the development of environmentally clean and efficient energy technologies (see Godø et al. 2003). These projects are organised as research consortia between R&D intensive firms including state-owned companies such as Statoil.

The objective of this paper is to explore the role of a public firm in influencing the structure of the network, and the potential implications of a public firm's presence for the relationship between equilibrium industry structure and performance, two key issue of the literature on R&D networks. The most natural way of studying which network architectures will materialise is to adopt Jackson and Wolinsky's (1996) concept of pairwise stability. It requires that a network is pairwise stable if no firm has an incentive to delete one of its existing links and no pair of firms want to establish a new link. Note that this condition is quite weak and thus should be seen only as a necessary condition for stability. Pairwise stability allows for deviations by a pair of firms. However, it could be the case that a group of firms can improve their competitive position by deleting or adding several links, which is not a possible deviation in the context of pairwise stable networks. To this end, Jackson and van den Nouweland (2005) introduced the concept of strong stability: we say that a network is strongly stable if it survives all possible deviations by a coalition of firms.[1]

In particular, we are primarily interested in the following questions:

(i) What are the incentives of competing firms that pursue efficiency-enhancing innovations to create networks for the purpose of sharing new knowledge? What is the architecture of the networks that will endogenously emerge?

(ii) How does the presence of a state-owned company affect the network structure; and, are individual incentives to form networks adequate from an efficiency point of view?

To answer these questions, we consider an environment with a public firm and two private firms. The timing of moves is as follows. In stage one, prior to competing in the product market, firms create collaboration ties. The purpose of collaboration agreements is the sharing of know-how about a cost-reducing technology. Six conceivable network structures arise from this stage. Under the *complete network* all firms are connected, whereas under the *empty network* there are no collaborative ties. A *star network* entails that there is a "hub" firm, either public or private, that maintains a direct link with two "spoke" firms, whereas the latter are indirectly connected via the "hub". Finally, under a *partial network* there is only a pair of firms with a collaborative link. In stage two, firms choose a non-cooperative level of R&D effort. A firm's own R&D effort together with the effort of its partners and the structure of the network determine its operating costs. In the last stage, firms compete in the market of a homogeneous good by choosing their quantities.

Our first result concerns the relationship between the level of collaborative activity and individual R&D effort. We find that even though R&D effort decreases with the number of alliances of a private firm, it increases with the number of alliances of the public firm.[2] To see intuitively why this happens, notice that the formation of a link implies two effects. On the one hand, as a firm establishes a link, it drives down its own production costs and expands its output. On the other hand, the costs of partner firms become lower too, which makes them tougher competitors. It turns out that this negative effect cannot be outweighed by the positive effect of collaboration on a firm's own quantity, thereby leading a private firm to exert a lower R&D effort. By contrast, both effects pull in the same direction when the public firm engages in collaborations and thus lead to a greater R&D effort. While this is so in the absence of R&D subsidies, it turns out that the provision of an R&D subsidy encourages not only the public firm but also the private firms to put in a higher effort when they engage in collaborations.

Our second result explores the stability properties of R&D networks. In particular, we show that the complete network is the unique pairwise stable and strongly stable network. This finding is mainly driven by the fact that the public firm is an aggressive competitor as it produces more output than a private firm. A higher output induces the private firms to form links in order to limit the competitive strength of the public firm. Thus our result can be interpreted in the following natural way: the stability of the complete network is not due to any enhancing effect of public ownership on the private firms' incentives to collaborate. Rather, it is due to the maximising behaviour of the public firm, which encourages collaboration by leaving a small residual demand to the private firms.

Our next result looks at the efficiency properties of R&D networks. In particular, we establish that the complete network is Pareto efficient, independently of the extent of technological spillovers. The private partial network and the private-hub star network are also Pareto efficient but within a smaller range of spillover values. More interestingly, when R&D subsidies are provided by the regulator, the complete network becomes the unique Pareto efficient network. Taken together, these results carry an important message: they suggest that a public firm can reconcile individual incentives for collaboration with the objective of efficiency, independently of whether R&D is subsidised.

[Insert Table 1 about here]

Our paper contributes to the growing literature on R&D networks. This literature focuses mainly on the analysis of the network architectures that will endogenously emerge and on the efficiency properties of the resulting networks. Goyal and Moraga-González (2001) do so in a setting with an arbitrary number of horizontally related firms and symmetric networks. They also analyse the three-firm case, which focuses on strategic incentives for collaboration by allowing firms to gain competitive advantages. Since this study it has been widely accepted that stable and efficient networks can differ. Goyal and Moraga-González (2001) find that such a conflict arises when spillovers are not too small. Mauleon, Sempere-Monerris, and Vannetelbosch (2008) provide a corresponding finding in a setting where each firm sets its own wage.

Song and Vannetelbosch (2007) investigate the possibility of resolving the potential conflict between stable and efficient networks by means of an R&D subsidisation policy. In particular, using a setting with three firms located in different countries and selling a (homogeneous) good within an internationally integrated product market, they show that the likelihood of a conflict is reduced but it is still present in the cases of very small or quite large spillovers.[3] In light of this, our result yields an insight into the role of a public firm as a policy instrument in regulating innovative activity. It suggests that a public firm can align individual incentives for collaboration with the objective of efficiency, independently of whether R&D subsidies are provided by the regulator. Other authors have reported similar conclusions though in a different context. In particular, Mauleon et al. (2008) find that the complete network emerges as the unique stable architecture when the labour market is unionised and wages are set at the firm-level. Also, the complete network maximises industry profits and so is the unique (strongly) efficient network.

Our paper also contributes to the R&D literature in mixed oligopolies. Delbono and Denicolò (1993) examine the role of a public firm in regulating innovative activity in a mixed duopoly with perfectly protected innovations. They show that a welfare-maximizing firm can alleviate the overinvestment problem in the private duopoly. Poyago-Theotoky (1998) investigates the case of easy imitation in R&D, showing that most of the results of Delbono and Denicolò (1993) can actually be reversed.[4] Our approach is richer in the sense that the strategic effects of the R&D are mediated through a network of R&D collaboration within which the place firms occupy and the structure of the network play an important role. This in turn may give us a more comprehensive view of how research incentives are shaped in the present context.[5]

The remainder of the paper is organised as follows. Section 2 presents the model.

The next section contains our results on the stability and efficiency properties of R&D networks. In section 4, we consider extensions of our model and, in section 5, we conclude. The equilibrium of the different network structures is characterised in the Appendix 1, and the proofs that are not included in the main text are relegated to the Appendix 2.

2 The model

We consider a model of endogenous network formation. Firms create collaboration links to transfer knowledge on a new technology which enhances their productive efficiency and hence, lowers costs. We study the incentives for R&D collaboration, paying particular attention to the form that strategic alliances can take, and then compare stable with efficient networks. We proceed first to develop the necessary terminology and definitions.

Networks of collaboration. Let $N = \{0, 1, 2\}$ be the set of firms. The set comprises a public firm (indexed by $i = 0$) and 2 identical private firms. The inverse demand function of the homogeneous good produced by the firms is $P(Q) = a - Q$, where $Q \leq a$ and $Q = \sum_{i \in N} q_i$. We will say that any two members of N, i and j, are linked under the network g if $\{i, j\} \in g$. For simplicity, we write ij to represent the link $\{i, j\}$, so that $ij \in g$ implies that firms i and j maintain a collaboration link under network g. Define a collaboration network as a collection of such *pairwise* links $\{(ij)_{i,j \in N}\}$. In any network g, nodes represent the firms and each link represents an R&D partnership. Firms can add or sever links from a given network.[6] We have that $g + ij$ is the network resulting from g if firms i and j form a new link between them. Similarly, $g - ij$ is the network resulting from the deletion of the link between i and j. Let $N_i(g)$ be the set of links of firm i in network g and let G be the set of all possible networks.

Within an industry consisting of three firms, we have the following network architectures: (i) the complete network, g^c, in which the level of collaborative activity is maximal, i.e. all firms are connected to each other; (ii) the star network, in which one firm ("hub") is connected with two others ("spokes"), while the latter are indirectly connected via the hub. Note that there are two cases to be distinguished here: either the public firm or any of the private firms can be a hub. We call the former public-hub star network, g^{s_0}, and the latter private-hub star network, g^s. (iii) Next we have the partially connected network, in which any two firms are connected while the third firm is isolated. Under the partially connected network either two private firms can maintain a link or the public firm can be linked with a private firm. We call the former private partial network, g^p, and the latter public partial network, g^{p_0}. (iv) Finally, we have the empty network (g^e), in which the level of collaborative activity is minimal, i.e. there are no collaboration links.

With two private firms and one public firm eight network architectures are possible; however, only six of them yield qualitatively different results. These network architectures are presented in Figure 1.

[Insert Figure 1 about here]

R&D efforts and spillovers. Given a network g, each firm carries out R&D to reduce its marginal cost. R&D effort is costly with cost represented by the quadratic function $\Gamma(e_i) = \gamma e_i^2$, $\gamma > 0$; this reflects diminishing returns to the level of R&D effort e_i. For simplicity, we set $\gamma = 1$ which ensures non-negativity of all variables.

Firms can attain a further reduction in their marginal costs by forming collaboration links. Given a network g and a collection of R&D efforts $\{e_i(g)\}_{i\in N}$, firm i's total effective R&D is given by

$$E_i(\{e_i(g)\}_{i\in N}) = e_i + \beta(\frac{e_j}{t(ij)} + \frac{e_k}{t(ik)}), \ \ i \neq j \neq k. \tag{1}$$

The effective level of R&D is the total reduction in a firm's marginal cost and has two components: the own research effort e_i and the effort profile of firm i's research partners $\{e_j, e_k\}$, $i \neq j \neq k$. We assume that the extent of information leakage or degree of spillovers benefit collaborating firms at an exogenously given rate β, $\beta \in (0, 1]$. The rate of knowledge transmission, the spillover rate, depends on the distance among collaborating firms. The distance between two firms i and j in a network g is defined as the number of links in the shortest path between them. We denote by $t(ij)$ the number of links in the shortest path between i and j, and we set $t(ij) = \infty$ to denote the absence of a path between them. Therefore, spillovers that result from direct collaborations are always larger than those obtained from indirect ones, since $t(ij) = 1$ in the case of a direct relationship.

The process of knowledge transmission is central to our analysis and so we further discuss the ideas underlying it. It follows Mauleon et al. (2008) and suggests that every collaborative agreement benefits from partial spillovers while there is no spillover outflow to non-collaborators. Goyal and Moraga-González (2001) focus instead on spillovers outside a given network. In particular, their formulation differs from ours in the following key respects.[7]

First, spillovers from direct collaborations are not fully absorbed. The assumption of partial spillovers reflects that knowledge comprises know-how that is firm-specific and thus cannot be easily absorbed and/or utilised by the research partners. This is also motivated by the growing complexity of new technologies and the implementation of distinctive processes within each firm. For example, if Sony and Philips decide to establish an agreement to reduce their costs for a DVD technology, they first need to set up common standards. This means that firms have to forgo short-term profit objectives anticipating to enhance their product-market positioning in the long-run.[8] However, in most cases, collaboration agreements are driven by both objectives at the same time, which justifies our focus on partial within-the-network spillovers (see Narula and Hagedoorn 1999). A related observation behind our assumption reflects the idea that firms need to build their absorptive capacity in order to benefit from technological spillovers (Cohen and Levinthal 1989). Because building absorptive capacity is costly, firms are able to incorporate only part of their partners' knowledge into their innovation process. In contrast, Goyal and Moraga-González assume that collaborating firms can fully benefit from each other's R&D efforts (i.e. there are perfect spillovers).

Second, empirical evidence suggests that the extent of knowledge transmission depends on the distance between collaborating firms.[9] As a way of capturing this empirical observation, we postulate that spillovers depend on the distance between collaborating firms and, in particular, they diminish with increasing distance between a pair research partners. Close in spirit to our approach, Piga and Poyago-Theotoky (2005) develop a Hotelling-type model to investigate firms' location decisions when R&D investment is directed towards enhancing product quality. In this paper, location decisions and thus, the distance – in a literal sense – between firms, have a bearing on the degree of knowledge dissemination. Finally, contrary to Goyal and Moraga-González, the present paper makes a clear distinction between directly and indirectly connected firms, although these authors treat both types of firms alike, assuming the same incoming spillover.

Payoffs. A network of collaboration is a collection of pairwise links in which the level of R&D effort and the extent of knowledge transmission depends on the place where firms locate in a given network. The cost of firm i in network g when firm i produces output q_i, and firm i's effective R&D output is E_i, is given by[10]

$$C_i(q_i, E_i, g) = (\bar{c} - E_i(g))q_i(g) + q_i^2(g), \quad i \in N, \ \ a > \bar{c} > 0. \tag{2}$$

Our specification of the cost function reflects the fact that all firms are ex ante equally efficient. If the public firm was more efficient it would serve the entire market; and if it was too inefficient this would leave room for potential privatisation.[11] We have further introduced a quadratic term in the firms' cost function to avoid situations where the private firms are driven out of the market altogether. Because our primary purpose is to study strategic interactions between a public and two private firms, this assumption is a natural way to do so by allowing for cost differences between the firms in equilibrium. This means that the public firm will incur a higher marginal cost, given that it seeks to maximise total surplus.[12]

As concerns private firms, they are assumed to maximise own profit

$$\pi_i(g) = [a - q_i(g) - q_j(g) - q_k(g)]q_i(g) - C_i(g) - e_i^2(g), \tag{3}$$

whereas the public firm maximises welfare defined as the sum of consumer surplus and producer profits

$$W(g) = \frac{Q^2(g)}{2} + \sum_{i=0}^{2} \pi_i(g). \tag{4}$$

The form of the public firm's objective function, placing equal weight on consumer and producer surplus, accords with utilitarianism or doctrines aimed at promoting fairness among consumers and producers. This is consistent with the purpose of this work which is to compare stable with efficient networks. In other words, we intend to examine the circumstances under which the presence of a public firm reconciles private incentives for collaboration with the objective of efficiency, which is a normative question. We thus restrict attention to an equally-weighted form of welfare function. Furthermore, we note that the assumption of welfare maximisation neglects any agency problems between the government and the public firm. However, this is an initial attempt to study R&D networks with a public firm, and in order to focus on strategic aspects of the interaction between the firms in this setting, maintaining this assumption provides a building block for the analysis of more general cases. We note that the literature on mixed oligopoly has extensively used similar assumptions (see e.g. Anderson et al. 1997; De Fraja and Delbono 1989; Pal and White 1998; White 2002; Fjell and Heywood 2004).

The timing of moves. We construct a three-stage game: in stage one, firms choose simultaneously and independently their collaborative links. In stage two, firms choose an individual level of R&D effort and, in the last stage, they compete in quantities.

The timing of moves reflects that a long-run decision, such as the formation of links, may have considerable effects on shorter-run decisions, such as the specific level of R&D and output. That is, when the firms decide which links to establish it is natural to anticipate how this may influence their R&D decisions and their product-market positioning. This timing which is standard in the R&D network literature also allows us to capture the commitment value of collaboration (see e.g. Narula and Hagedoorn 1999). To solve this multi-stage game, we first obtain the equilibrium of stages two to three by backward induction. Finally, we solve stage one, the network formation stage, by applying the notion of pairwise stability.

3 Network formation

R&D EFFORTS

We begin our analysis by addressing the following question: What is the impact of forming links on the firms' R&D effort? Would an increase in the number of strategic alliances increase own R&D effort or would it induce a reduction in the effort due to free-riding? Our notational convention is as follows. The superscript l refers to a linked firm in a partially connected network and the superscript h denotes the "hub" in a star network. The superscripts e, p_0, p, s_0, s, c stand for the empty, public partial, private partial, public-hub star, private-hub star and complete networks. Proposition 1 reports how changes in the level of collaborative activity affect the firms' R&D effort.

Proposition 1 *The public firm's R&D effort:*

(i) Increases with the number of own links.

(ii) Increases with the degree of R&D spillovers unless a private partial network is formed.

A private firm's R&D effort:

(iii) Decreases with the number of own links except in the move from the empty network to the public partial network if R&D spillovers are sufficiently large.

(iv) Decreases with the degree of R&D spillovers except under a public partial network and a public-hub star network where it is non-monotone and achieves a minimum when β *is approximately equal to 0.46 and 0.62, respectively.*

Proof. (i) Follows directly from the comparison $e_0(g^e) < e_0^l(g^{p_0}) < e_0^h(g^{s_0})$. (ii) Follows by differentiating the public firm's R&D effort in the various network architectures. (iii) Follows from the comparisons $e(g^e) < e^l(g^{p_0}) < e^h(g^s)$ and $e(g^e) < e^l(g^p) < e^h(g^s)$. From Tables 7 and 9, we have $e(g^e) \leq e^l(g^{p_0})$ if and only if $\beta \geq 0.92$ and $e(g^e) > e^l(g^{p_0})$, otherwise. (iv) Follows by differentiating each private firm's R&D effort in the various network architectures. ■

Result (i) states that when the public firm forms additional links, it finds particularly appealing to increase its R&D effort. This highlights the combined effect underlying direct and indirect spillovers. Specifically, in the move from the empty to the public partial network, the public firm can benefit from direct spillovers. By moving then to the public-hub star network, the public firm can appropriate technological know-how not only through direct but also indirect links. Intuitively, the efficiency effect of adding a link is to increase the public firm's output by reducing own production costs. A higher output makes further increases in R&D effort more worthwhile because the potential gains from higher efficiency will spread across more units of output.

[Insert Figure 2 about here]

By result (ii) these effects are more pronounced when the spillover rate within a given network increases because of the greater potential for inter-firm communication and learning. In contrast, the private firms' move from the empty to the private partial network reduces the public firm's market share and thereby reduces its R&D effort by leading to a lower level of public firm output. The negative effect of remaining isolated becomes more pronounced as the spillover rate increases, which produces the lower effort of the public firm under a private partial network, as Proposition 1 reports.

Result (iii) states that a private firm exerts a lower effort when it links with other firms. An increase in the number of links reduces own production costs because a firm can benefit from the investments of its partners. The addition of links also lowers production costs of partner firms and thereby increases the intensity of product market competition. The resulting increase in the intensity of competition outweighs the corresponding increase in own efficiency. Consequently, R&D effort is lower when a private firm forms research partnerships (see Goyal and Moraga-González 2001). In contrast, in the move from the empty network to the public partial network, a private firm increases its R&D effort when spillovers are sufficiently large, because it can reduce the market share and thereby limit the competitive strength of the isolated firm.

Result (iv) refers to the role of spillovers for the level of the private firms' R&D effort. When the rate of spillover increases, partner firms become more aggressive competitors. The resulting increase in the intensity of competition decreases R&D effort because a private firm will have an incentive to free ride on its partners' research efforts. An exception to this rule is again the public partial network. In this case, a higher effort increases the output of the linked firms and thereby reduces the output of the isolated firm. Therefore, a private firm will put in a higher effort when spillovers increase beyond as certain threshold, as part (iv) of Proposition 1 states. Furthermore, within a public-hub star network a private firm realises that by increasing its own R&D effort it can enhance the public firm's payoff because the public firm takes into account private profit. The public firm benefits from greater private effort and thereby increases its own R&D effort, which benefits the private firm. Consequently, a private firm will increase its effort under a public-hub star network when spillovers increase beyond a certain level.

STABILITY AND EFFICIENCY

PAIRWISE STABILITY

R&D alliances are conceptualised in terms of pairwise links which are embedded in a more general context of bilateral relations – a network. Therefore, to address the issue of network formation, one can use the definition of pairwise stability to examine which network architectures will endogenously emerge. The following definition is due to Jackson and Wolinsky (1996) and refers to the firms' incentives to alter the structure of a network by creating or severing bilateral links. This definition is quite weak and should therefore be seen only as a necessary condition for stability (Jackson and Wolinsky 1996; Goyal and Moraga-González 2001).

Definition 1 *A network g is pairwise stable if the following conditions are satisfied:*

(i) If firms i, $j \in N$ are private

(a) for all $ij \in g$, $\pi_i(g) \geq \pi_i(g - ij)$ and $\pi_j(g) \geq \pi_j(g - ij)$, and

(b) for all $ij \notin g$, if $\pi_i(g) < \pi_i(g + ij)$, then $\pi_j(g) > \pi_j(g + ij)$.

(ii) If firm i is public and firm j is private

(a) for all $ij \in g$, $W(g) \geq W(g - ij)$ *and* $\pi_j(g) \geq \pi_j(g - ij)$, *and*

(b) for all $ij \notin g$, *if* $W(g) < W(g + ij)$, *then* $\pi_j(g) > \pi_j(g + ij)$ *(and vice versa).*

The definition of Jackson and Wolinsky (1996) is adapted to allow for a public firm as a member of a network. In the absence of a public firm, definition 1 reduces to conditions $i(a)$ and $i(b)$ that have been used in related papers by Goyal and Moraga-González (2001), Song and Vannetelbosch (2007), Mauleon et al. (2008), among others. Definition 1 says that a network is pairwise stable if it survives all possible deviations at a bilateral level, that is, if no firms have an incentive to delete one of their links, and no pair of firms want to form a new link with one benefiting strictly and the other at least weakly (see Jackson and Wolinsky 1996; Mauleon et al. 2008). Thus, joint consent is required in order to establish a bilateral relationship (i.e. a link cannot be enforced), and a link can be simply deleted unilaterally. We apply this definition to study pairwise stable networks.

Proposition 2 *In the presence of a public firm, the unique pairwise stable collaboration network is the complete network.*

It appears that in the empty, partial and star networks firms that are not connected have an incentive to establish a collaboration. Interestingly, in the private partial network g^p, in which there is a collaborative agreement between the private firms whereas the public firm is isolated, it turns out that each private firm has an incentive to set up a new link with the public firm in order to become the "hub" in the ensuing private-hub star network g^s. Because the public firm is better off by engaging in collaboration with a private firm, the private partial network is destabilised, thus giving rising to the private-hub star network, i.e. $W(g^s) > W(g^p)$ and $\pi^h(g^s) > \pi^l(g^p)$ – see Figures 3 and 4.

This is in contrast with the outcome in a purely private market. In this case, the partial network remains stable for small spillovers if R&D is not subsidised (Goyal and

Moraga-González 2001). When R&D subsidies are available, though, the partially connected network becomes stable for intermediate spillovers (Song and Vannetelbosch 2007). The intuition behind these results stems from the large disparities between the linked firms and the isolated one in a partially connected network, which make (in extreme cases) the isolated firm to exit the market. By contrast, Proposition 2 indicates that when a public firm is isolated, then each linked firm has an incentive to establish a connecting link with it. The reason is that the public firm invests in R&D more than a private firm, so that setting up a link with it enables a private firm to increase its payoff through direct spillovers both from the public firm and from its current partner. This, in turn, destabilises the private partial network g^p, leading to the private-hub star network g^s.

By analogous reasoning, in the private-hub star network, the private firm at one of the spokes has an incentive to establish a new link with the public firm; this gives rise to the complete network given that the public firm always benefits from having an additional collaboration, i.e. $W(g^c) > W(g^s)$ and $\pi(g^c) > \pi(g^s)$ – again, see Figures 3 and 4.

Next, consider the public-hub star network g^{s_0}. In this case, the private firms have an incentive to link to each other in order to limit the public firm's competitive strength, i.e. $\pi(g^c) > \pi(g^{s_0})$. This in turn destabilises the public-hub star network, giving rise to the complete network. Thus the presence of a public firm increases the degree of partnering intensity, so that the complete network becomes the unique pairwise stable architecture. It is noteworthy that the stability of the complete network is not the outcome of any enhancing effect of public ownership on the private firms' incentives to collaborate. Rather, it is due to the maximising behaviour of public firm which reduces the market share and so reduces the profits that accrue to the private firms. Therefore, the private firms have an incentive to establish new collaborations in order to offset the negative effect on their profits resulting from the public firm's behaviour. This result is also consistent with the case in which all firms are private because the complete network is always pairwise stable then.

Our final observation concerns the role of spillovers in the stability of the complete network. Note that the relevant network architectures become more prominent when spillovers are relatively large. By contrast, in the limiting case that spillovers tend to zero, the network architectures become very similar. Consequently, when spillovers become smaller, this leads to a decrease of the (potential) losses from deleting a link from the complete network.

STRONG STABILITY

We proceed to perform an additional check for stability by resorting to the notion of strong stability due to Jackson and van den Nouweland (2005). This notion of stability refers to the incentives of a coalition of firms to redistribute their collaboration links, and so it allows for situations which are not accounted for under pairwise stability. Indeed, we will say that a network g is strongly stable when it survives all possible changes in the number of its links by a coalition of agents. Because strong stability is a refinement of pairwise stability, the only candidate for strongly stable network is the complete network. Possible deviations from the complete network are the following: (i) the coalition of the two private firms deleting their links with the public firm to form the private partial network, g^p; (ii) the coalition of the public firm and one private firm severing their link with the other private firm to form the public partial network, g^{p_0}; and the coalition of all firms deleting their connecting links to establish the empty network, g^e. It turns out none of these deviations increases the payoff of the coalition of agents attempting to alter the structure of the complete network. Consequently, the complete network emerges endogenously as the unique strongly stable network.

This result sharply contrasts with the outcome in a purely private market. When all firms are profit-maximisers and the government does not subsidise R&D, the partially connected network is the unique strongly stable network if spillovers are sufficiently small (Song and Vannetelbsoch 2007). When R&D is subsidised, though, the partially connected network remains stable against deviations by a coalition of firms if spillovers obtain intermediate values (Song and Vannetelbsoch 2007). In contrast, our analysis shows that the partially connected network is no longer pairwise stable in a mixed market, and so it cannot be strongly stable too. Firms have instead incentives to connect to each other in order to form the complete network.

[Insert Figure 3 about here]

EFFICIENCY

In this section, we examine the efficiency properties of the R&D networks. Such analysis is important to understand the relationship between stable and efficient networks, a key issue of the network literature in oligopolistic industries.

Pareto efficiency is a natural notion of efficiency: we will say that a network $g \in G$ is Pareto efficient if it is not Pareto dominated by any other network. That is, if there does not exist $g' \in G$ such that $W(g') \geq W(g)$ and $\pi_i(g') \geq \pi_i(g)$ for all $i \in \{1, 2\}$, and with strict inequality for some firm either public or private. Application of this definition reveals that only three networks architectures are Pareto efficient. Importantly, the complete network is the unique Pareto efficient network when spillovers are sufficiently large. The following Proposition elaborates.

Proposition 3 *The complete network g^c is always Pareto efficient, the private-hub star network g^s is Pareto efficient if and only if spillovers are not too large ($\beta < 0.82$) and the private partial network g^p is Pareto efficient if and only if spillovers are sufficiently small ($\beta < 0.13$). The public-hub star network g^{s_0}, the public partial network g^{p_0} and the empty network g^e are never Pareto efficient.*

Taken together with our findings on the stability properties of the different networks, the present result carries an important message: it suggests that the presence of a public firm can reconcile individual incentives to form R&D networks with the objective of efficiency. In contrast, a conflict between stable and efficient networks within a private market is likely to occur when spillovers are relatively large and there are no subsidies to R&D. However, when subsidies to R&D are available, such a conflict is likely to arise when spillovers are very small or quite large. The present paper investigates the possibility of resolving the conflict between stable and efficient networks that is likely to arise in a purely private market, thus highlighting the role of a public firm as a policy instrument in this setting. Although this result has been derived within a rather limited context, it can be thought of as a building block that could guide future research aimed at improving our understanding of the circumstances under which privatisation programs should (not) be allowed because of the potential adverse consequences they might have on the formation of R&D networks that are of optimal size from an efficiency point of view.

[Insert Figure 4 about here]

4 Discussion and extensions

Our model is rather stylised so it is natural to check the robustness of our results. In what follows, we extend our analysis in two main directions: R&D subsidies and the number of firms.

R&D SUBSIDIES

The analysis to this point has abstracted from R&D subsidies. However, the role of an R&D subsidy is to address fundamental market failures that generate suboptimal investment levels. In this section we assume that after firms have decided their collaboration links, at stage two of the game, the regulator provides a subsidy per unit of R&D effort. The regulator maximises welfare defined as the sum of consumer surplus and producer profits net of R&D subsidies

$$W(g) = \frac{Q^2(g)}{2} + \sum_{i=0}^{2} \pi_i(g) - s[e_i(g) + e_j(g) + e_k(g)], \quad i \neq j \neq k.$$

Because as in Goyal and Moraga-González (2001), Deroian and Gannon (2006), Goyal et al. (2008), Mauleon et al. (2008) there are no costs associated with the formation of links, the complete network corresponds to the first best. However, even in the complete network an R&D subsidy is needed for three main reasons. First, there is underinvestment due to imperfect competition. Second, a private firm does not take into account consumer surplus in its objective and thus chooses a lower level of R&D relative the social optimum – so-called undervaluation effect (Ulph 1999). Third, the objective of the public firm, being consistent with welfare maximisation, takes into account consumer surplus. In doing so, the public firm introduces another type of market failure – inefficiency in production – which is related to the composition of R&D. A further source of market failure arises due to the fact that firms do not fully share the outcomes of their research. Therefore, the role of an R&D subsidy is two-fold in this setting. It remedies the suboptimal level of R&D investment by encouraging firms to spend more in R&D. In addition to this, the subsidy has a cost shifting effect that helps improve the distribution of production costs between the public and the private firms, thereby increasing productive efficiency.

In the working paper version of this article (Zikos 2008), an extensive analysis of the subsidy case is provided. Thus our discussion here focuses on the most interesting results. We find that R&D effort increases when a private firm establishes new links for three primary reasons. First, a private firm reduces its own costs by forming collaborations. This direct effect works toward improving a firm's competitive position. Second, the formation of new collaborations reduces the costs of partner firms and thereby makes them stronger competitors. As explained above, the loss a firm suffers from the increase in the competitiveness of its partners outweighs the positive effect of lowering own costs. Therefore, a private firm reduces its R&D effort when it engages in new partnerships (recall Proposition 1). However, in the presence of R&D subsidies, there is a potential countervailing effect: an increase in a private firm's R&D effort increases the size of the R&D subsidy it receives. It turns out that the combined influence of the subsidy and the reduction in a firm's own costs dominate the increase in the competitiveness of partner firms. Consequently, a private firm exerts a higher R&D effort when it engages in new research collaborations. In addition to this, it can be shown that a private firm increases its R&D effort when spillovers become higher in this setting with R&D subsidies.

We also find that the complete network is the unique pairwise stable and strongly stable network. More interestingly, the complete network is the unique Pareto efficient network. Thus, a public firm can reconcile the conflict between stability and efficiency that is likely to arise in a purely private market, which reinforces the result of our basic model.

FOUR FIRMS

The analysis to this point has considered an industry consisting of three firms. To assess the effects of the number of firms most simply, consider a setting with three private firms and one public firm. While acknowledging that the most realistic scenario would involve asymmetric network structures, following Goyal and Moraga-González (2001) we cast our analysis in the context of symmetric networks because our primary objective is to further explore the stability and efficiency properties of the complete network (see Figure 5). A symmetric network of degree k is denoted by g^k, for $k \in \{0, 1, 2, 3\}$. The degree of a network (or level of collaborative activity) is defined as the number of links of firm i,

$i \in \{0, 1, 2, 3\}$.

In this environment, the key qualitative predictions of our basic model persist. In particular, the complete network g^3 is the unique stable network when a public firm competes with three private firms. The complete network is in addition the unique Pareto efficient network. Thus, as Proposition 4 reports, a public firm can reconcile individual incentives for collaboration with the objective of efficiency in this setting.

Proposition 4 *Within an industry consisting of a public and three private firms, the complete network is uniquely stable and efficient in the class of symmetric networks.*

This result yields interesting insights into the role that a public firm can play in influencing the structure of a network and its effect on the relationship between stable and efficient networks.[13] We emphasise that care should be taken when generalising this result to markets with many firms, both public and private. However, the simple setting employed here allows us to draw conjectures about the network architectures that one might expect to emerge in a more general setting. First, the empty network cannot be stable since any two firms have an incentive to establish a new connecting link. Second, it might appear that most of the collaborative alliances are formed between the public firm and private firms. This is because the public firm invests a larger amount in R&D than a private firm. Therefore, we would expect to observe networks having the public firm as a central node. Third, recall that the presence of a public firm reduces the asymmetries between the linked firms and the isolated one in a three-firm oligopoly. In a more general setting, we would expect to observe that the smaller the number of private firms, the stronger is the influence of the public firm in reducing the competitive advantage of firms with a large number of links relative to the firms with a smaller number of links, thus making the network structure more symmetric. Put differently, in an industry with a large number of public firms, we would expect to observe networks that consist of a relatively large number of links. These conjectures also present hypotheses that could be empirically tested.

[Insert Figure 5 about here]

5 Conclusion

A well-known result is that under a wide set of circumstances private firms underinvest in R&D due to a lack of full appropriability of the returns to their R&D. Previous authors examined the role of a public firm in regulating innovation activity when firms are independent competitors. Our approach extended these studies by offering a more comprehensive view of innovation activity as it allowed the strategic effects of R&D to be mediated through a network of R&D collaboration. The main novelty of our approach is that firms' strategic incentives to invest in R&D are shaped within a network of collaboration where they are embedded. Our paper also contributes to the literature on R&D networks, which has studied extensively the incentives of private firms to form collaborative alliances. This literature has recognised that individual incentives for collaboration need not always be aligned with the corresponding incentives from an efficiency viewpoint.

We have shown that, in the absence of R&D subsidies, the complete network is the unique stable network. The stability of the complete network is not the outcome of any enhancing effect of public ownership on the private firms' incentives to collaborate. Rather, it is due to the fact that the public firm is an aggressive competitor and as such it leaves a small residual demand to the private firms. Therefore, by forming additional links, the private firms can at least partially counter the depressing effect on their profits resulting from the public firm's maximising behaviour. Among other networks, we have demonstrated that the complete network is Pareto efficient. However, when R&D subsidies are available, the complete network is not only stable but also it is the unique Pareto efficient network.

What are the policy implications of the analysis? A public firm can be used as a policy instrument in tackling the conflict between stable and efficient networks that is likely to arise in a private market. However, we believe that the role of a public firm in restoring the 'correct' incentives for R&D collaboration would be more prominent the smaller the size and/or competitiveness of the relevant industry. On one argument, the fact that a public firm encourages collaborations and thereby promotes R&D spending helps to alleviate the so-called underinvestment problem. But this introduces another type of market failure that stems from the fact that the distribution of production costs is not efficient. Thus, a public firm may be a useful policy instrument, although with certain limitations. A future promising research direction is to empirically investigate the relationship between network architectures and the presence of public firms.

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Notes

[1]These definitions have been adapted to allow for a public firm as a member of a network.

[2]A natural exception to this occurs in the move from the empty to the public partial network. In this case, a private firm increases its R&D effort when the level of technological spillovers between collaborating firms is sufficiently high.

[3]Related is the finding that governments should be allowed to subsidise R&D whenever spillovers are not too small.

[4]Nett (1994) considers the case of a mixed duopoly with cost reducing innovation, and shows that the public firm may opt for producing at a higher cost than the private firm. Moreover, under certain circumstances, welfare can be higher in the private duopoly than in the mixed duopoly.

[5]Powell, Koput, and Smith-Doerr (1996) mention that the locus of innovation is not anymore a firm as a single entity. Rather, it is the network of collaboration where the firm is embedded.

[6]The optimality of forming or severing links within a network is conceptualised in terms of profits for the private firms whereas in terms of welfare (consumers surplus plus aggregate profits) for the public firm.

[7]The same approach as in Goyal and Moraga-González (2001) is adopted by Song and Vannetelbosch (2007).

[8]Establishing common standards requires that firms incur an investment cost, which can be recovered in the long-run.

[9]The empirical literature to date is a bit less clear on this issue. For example, existing findings suggest that the probability of collaboration between private firms and universities depends negatively on their physical distance (see eg. Mansfield and Lee 1996; Anselin et al. 1997). Cast in this light, knowledge externalities between private firms and universities are to a large extent geographically clustered (e.g. Silicon Valley in California or Waterloo region in Ontario).

[10]Note that the marginal cost of production is given by $c_i(g) = \frac{\partial C_i}{\partial q_i} = (\bar{c} - E_i) + 2q_i$. Therefore, the impact of effective R&D on the margin is to induce a downward shift in each firm's cost curve, without affecting its slope. This specification, in a simple way, maintains the spirit of earlier contributions (see

e.g. d' Aspremont and Jacquemin 1988; Goyal and Moraga-González 2001; Song and Vannetelbosch 2007; Mauleon et al. 2008).

[11]White (2002) points out that this assumption can be qualified in several ways. For instance, there is mixed evidence on the relative efficiency of public and private firms, so that assuming that the public firm is (ex ante) as efficient as the private firms would seem quite reasonable. Furthermore, public firms that survive for a significant time period may fall within the same category of being relatively efficient.

[12]In the absence of R&D subsidies, the public firm can make a loss in equilibrium for specific values of the spillover parameter. This is consistent with a number of articles that have appeared lately in the press according to which about 32.4% and 36.2% of the state-owned industrial firms in China and the U.S. respectively reported losses at the end of 2005 (see e.g. Windle 2006). This also provides a rational for why public firms are subsidised in the real world. The implication of R&D subsidies for our results is analysed in section 4.

[13]Allowing for asymmetric networks, it can be shown that the complete network still emerges as the unique stable architecture. However, in addition to the complete network, other networks can be Pareto efficient due to strategic advantages that can arise in this setting. Namely, the network ij, $i \neq j$, $i, j \in \{1, 2, 3\}$, is Pareto efficient if $\beta < 0.087$; the network $g^3 \backslash 0i$, $i \in \{1, 2, 3\}$, is Pareto efficient if $\beta < 0.89$; and the network $g^3 \backslash ij$, $i \neq j$, $i, j \in \{1, 2, 3\}$, is Pareto efficient for all values of the spillover parameter.

Table

No subsidies	conflict if spillovers sufficiently large
R&D subsidies	conflict if spillovers small or quite large
(R&D subsidies) and state-owned firm	no conflict[a]

Table 1: Potential conflict between stable and efficient networks ([a]new result)

Figure captions

Figure 1: Six possible network architectures

Figure 2: R&D efforts of the public (left panel) and private firms (right panel) when adding own links

Figure 3: Welfare levels

Figure 4: Profits of private firms (left panel); illustration of intersection points for low spillovers (right panel)

Figure 5: Symmetric networks with four firms

Figures

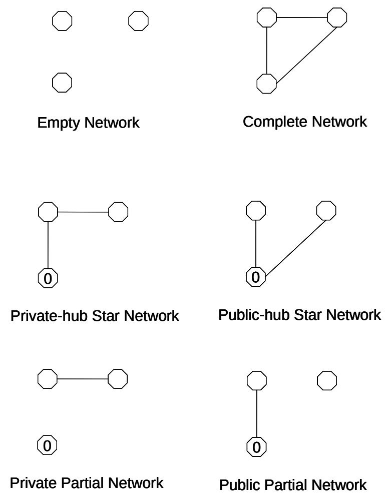


Figure 1

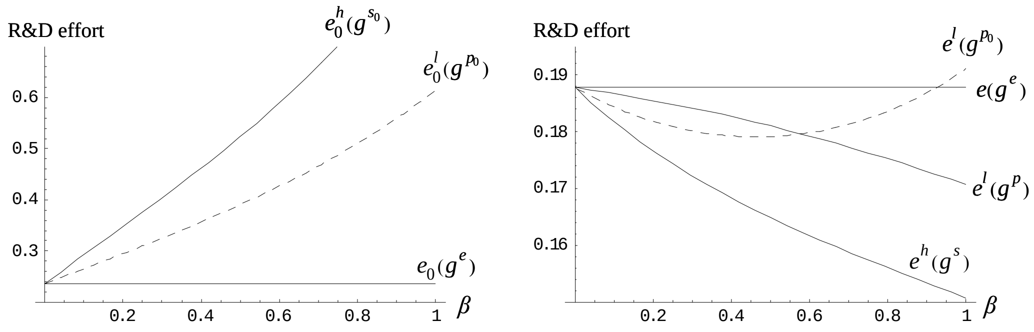

R&D effort
0.6
0.5
0.4
0.3
0.2
0.4
0.6
0.8
1
β
$e_0^h(g^{s_0})$
$e_0^l(g^{p_0})$
$e_0(g^e)$
R&D effort
0.19
0.18
0.17
0.16
0.2
0.4
0.6
0.8
1
β
$e^l(g^{p_0})$
$e(g^e)$
$e^l(g^p)$
$e^h(g^s)$

Figure 2

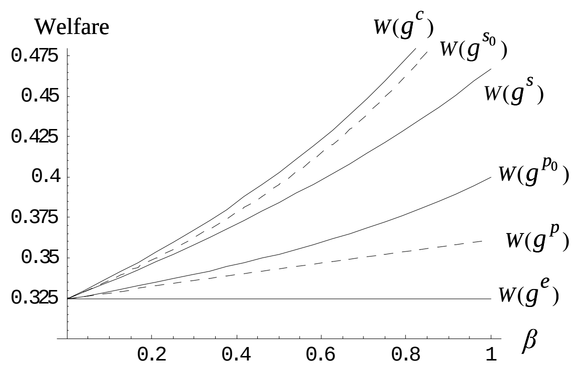
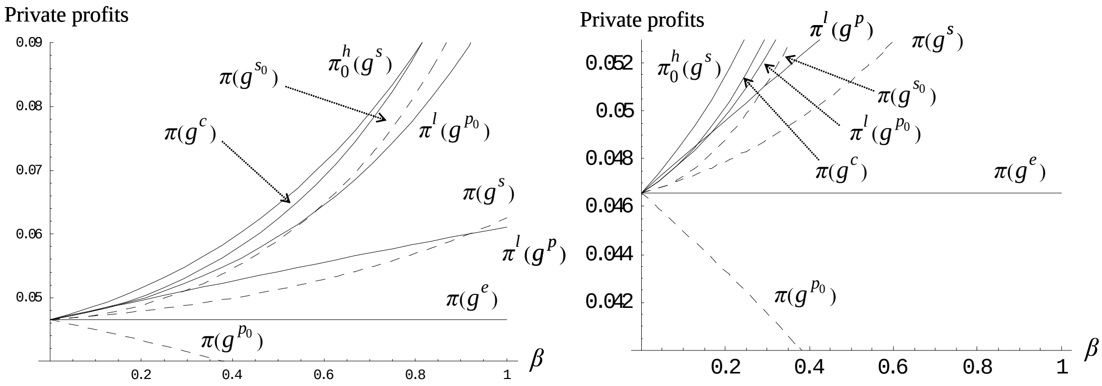

Welfare
0.475
0.45
0.425
0.4
0.375
0.35
0.325
0.2
0.4
0.6
0.8
1
β
W(g^c)
W(g^{s_0})
W(g^s)
W(g^{p_0})
W(g^p)
W(g^e)

Figure 3



Private profits
0.09
0.08
0.07
0.06
0.05
0.2
0.4
0.6
0.8
1
β
$\pi_0^h(g^s)$
$\pi(g^{s_0})$
$\pi(g^c)$
$\pi^l(g^{p_0})$
$\pi(g^s)$
$\pi^l(g^p)$
$\pi(g^e)$
$\pi(g^{p_0})$
Private profits
0.052
0.05
0.048
0.046
0.044
0.042
0.2
0.4
0.6
0.8
1
β
$\pi_0^h(g^s)$
$\pi^l(g^p)$
$\pi(g^s)$
$\pi(g^{s_0})$
$\pi^l(g^{p_0})$
$\pi(g^c)$
$\pi(g^e)$
$\pi(g^{p_0})$

Figure 4

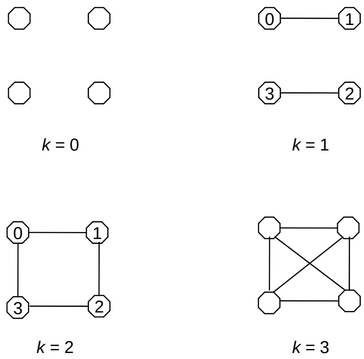


Figure 5

Appendix 1. Equilibrium outcomes

Networks with three firms

A.1. The complete network

In the complete network, g^c, all firms are connected. The marginal cost of firm i is given by $c_i(g^c) = \bar{c} - e_i - \beta(e_j + e_k) + 2q_i$, $i \neq j \neq k$. Standard calculations reveal the equilibrium R&D efforts, quantities, profits and welfare

Table 1: Complete network

$e_0(g^c) = (a-\bar{c})(83 + 224\beta - 10\beta^2)/A$	$e(g^c) = 2(a-\bar{c})(3+\beta)(11-5\beta)/A$
$q_0(g^c) = 2(a-\bar{c})(92 + 83\beta - 58\beta^2)/A$	$q(g^c) = 39(a-\bar{c})(3+\beta)/A$
$\pi_0(g^c) = 3(a-\bar{c})^2 B/A^2$	$\pi(g^c) = 2(a-\bar{c})^2(1279 + 220\beta - 50\beta^2)/A^2$
$CS = 2(a-\bar{c})^2(209 + 122\beta - 58\beta^2)^2/A^2$	$W(g^c) = (a-\bar{c})^2 C/A^2$

where $A = 703 + 220\beta - 322\beta^2 + 20\beta^3$, $B = 8989 + 7968\beta - 21216\beta^2 - 11344\beta^3 + 4452\beta^4$ and $C = 160373 + 164512\beta - 73772\beta^2 - 62656\beta^3 + 19884\beta^4$.

A.2. The private-hub star network

In the private-hub star network, g^s, a private firm is at the hub and is connected with the other two firms. Without loss of generality, let firm 1 be the hub. As for the spoke firms, each has a collaboration link with the hub and there is no direct link among them, although they are indirectly connected via the hub. The marginal cost structures are given by

$$
\begin{aligned}
c_0(g^s) &= \bar{c} - e_0 - \beta e_1 - (\beta/2)e_2 + 2q_0; \quad c_1(g^s) = \bar{c} - e_1 - \beta(e_0 + e_2) + 2q_1; \\
c_2(g^s) &= \bar{c} - e_2 - \beta e_1 - (\beta/2)e_0 + 2q_2.
\end{aligned}
$$

Equilibrium outcomes are readily shown to be the following

Table 2: Private-hub star network

Private hub firm
$e^h(g^s) = 2(a-\bar{c})(11-5\beta)(1140+716\beta+102\beta^2-21\beta^3)/D$
$q^h(g^s) = 39(a-\bar{c})(1140+716\beta+102\beta^2-21\beta^3)/D$
$\pi^h(g^s) = 2(a-\bar{c})^2(1279+220\beta-50\beta^2)(1140+716\beta+102\beta^2-21\beta^3)^2/D^2$

Table 3: Private-hub star network

Public spoke firm
$e_0(g^s) = 2(a-\bar{c})(15770+36128\beta+7419\beta^2-2876\beta^3+70\beta^4)/D$
$q_0(g^s) = (a-\bar{c})(69920+66338\beta-19336\beta^2-10829\beta^3+1769\beta^4)/D$
$\pi_0(g^s) = (a-\bar{c})^2F/D^2$
$CS(g^s) = (a-\bar{c})^2(158840+117116\beta-25186\beta^2-12428\beta^3+1769\beta^4)^2/2D^2$
$W(g^s) = (a-\bar{c})^2L/2D^2$

Table 4: Private-hub star network

Private spoke firm
$e(g^s) = 2(a-\bar{c})(22-7\beta)(570+293\beta-126\beta^2-10\beta^3)/D$
$q(g^s) = 78(a-\bar{c})(570+239\beta-126\beta^2-10\beta^3)/D$
$\pi(g^s) = 4(a-\bar{c})^2(2558+308\beta-49\beta^2)(570+293\beta-126\beta^2-10\beta^3)^2/D^2$

where $D = 267140+139916\beta-85504\beta^2-18595\beta^3+5987\beta^4-280\beta^5$ and $F = 3894034800+4718797440\beta-4460126572\beta^2-5861183392\beta^3-213254968\beta^4+803947604\beta^5+11616329\beta^6-36702442\beta^7+3109761\beta^8$ and $L = 46315722400+63774360320\beta+1297032744\beta^2-22755859920\beta^3-2286897492\beta^4+2699074464\beta^5+95329018\beta^6-116871708\beta^7+9221483\beta^8$.

A.3. The public-hub star network

In the public-hub star network, the public firm (hub) maintains a direct link with each private firm (spoke); the private firms, in turn, are directly connected with the public firm and indirectly connected with each other. The relevant cost structures under g^{s_0} are thus given by

$$c_0(g^{s_0}) = \bar{c} - e_0 - \beta(e_i + e_j) + 2q_0;$$

$$c_i(g^{s_0}) = \bar{c} - e_i - \beta(e_0 - (1/2)e_j) + 2q_i, \;\; i \neq j, \, i, j \in \{1, 2\}.$$

The equilibrium outcomes are as follows

Table 5: Public-hub star network

Public hub firm
$e_0^h(g^{s_0}) = (a - \bar{c})(83 + 233\beta - 12\beta^2)/\Theta$
$q_0^h(g^{s_0}) = 8(a - \bar{c})(23(1 + \beta) - 15\beta^2)/\Theta$
$\pi_0^h(g^{s_0}) = 3(a - \bar{c})^2(8989 + 9678\beta - 20867\beta^2 - 12856\beta^3 + 4752\beta^4)/\Theta^2$
$CS(g^{s_0}) = 2(a - \bar{c})^2(209 + 131\beta - 60\beta^2)^2/\Theta^2$
$W(g^{s_0}) = (a - \bar{c})^2(160373 + 175582\beta - 70251\beta^2 - 70072\beta^3 + 21328\beta^4)/\Theta^2$
Private spoke firms
$e(g^{s_0}) = 2(a - \bar{c})(3 + \beta)(11 - 4\beta)/\Theta$
$q(g^{s_0}) = 39(a - \bar{c})(3 + \beta)/\Theta$
$\pi(g^{s_0}) = 2(a - \bar{c})^2(3 + \beta)^2(1279 + 176\beta - 32\beta^2)/\Theta^2$

where $\Theta = 703 + 265\beta - 344\beta^2 + 16\beta^3$.

A.4. The private partial network

In the private partial network, g^p, there is a research collaboration between the two private firms while the public firm remains outside this collaboration. The relevant (marginal) cost structures are given by

$$c_0(g^p) = \bar{c} - e_0 + 2q_0, \quad c_1(g^p) = \bar{c} - e_1 - \beta e_2 + 2q_1$$

$$c_2(g^p) = \bar{c} - \beta e_1 - e_2 + 2q_2.$$

Equilibrium outcomes are the following

Table 6: Private partial network

Private linked firms
$e^l(g^p) = 6(a-\bar{c})(11-2\beta)/H$
$q^l(g^p) = 117(a-\bar{c})/H$
$\pi^l(g^p) = 18(a-\bar{c})^2(1279+88\beta-8\beta^2)/H^2$
Public isolated firm
$e_0(g^p) = (a-\bar{c})(83-18\beta+4\beta^2)/H$
$q_0(g^p) = 4(a-\bar{c})(46-9\beta+2\beta^2)/H$
$\pi_0(g^p) = 3(a-\bar{c})^2(8989-3420\beta+1084\beta^2-144\beta^3+16\beta^4)/H^2$
$CS\,(g^p) = 2(a-\bar{c})^2(209-18\beta+4\beta^2)^2/H^2$
$W(g^p) = (a-\bar{c})^2(160373-22140\beta+6956\beta^2-720\beta^3+80\beta^4)/H^2$

where $H = 703 - 90\beta + 20\beta^2$.

A.5. The public partial network

In the public partial network, g^{p_0}, the public firm and a private one maintain a single collaborative agreement while the remaining private firm stays isolated. Suppose that the collaboration link is among the public firm $j = 0$ and the private firm $i = 1$, without loss of generality. This generates the following (marginal) cost structures

$$\begin{aligned} c_0(g^{p_0}) &= \bar{c} - e_0 - \beta e_1 + 2q_0, \qquad c_1(g^{p_0}) = \bar{c} - e_1 - \beta e_0 + 2q_1 \\ c_2(g^{p_0}) &= \bar{c} - e_2 + 2q_2. \end{aligned}$$

Equilibrium outcomes are as follows

Table 7: Public partial network

Linked firms
$e_0^l(g^{p_0}) = (a-\bar{c})(7885 + 11744\beta - 594\beta^2)/J$
$e^l(g^{p_0}) = 2(a-\bar{c})(3135 + 124\beta - 311\beta^2 + 12\beta^3)/J$
$q_0^l(g^{p_0}) = (a-\bar{c})(17480 + 10147\beta - 5968\beta^2 - 26\beta^3 + 12\beta^4)/J$
$q^l(g^{p_0}) = 39(a-\bar{c})(285 + 89\beta - 4\beta^2)/J$
$\pi_0^l(g^{p_0}) = 81(a-\bar{c})^2K/J^2$
$\pi^l(g^{p_0}) = 2(a-\bar{c})^2(285 + 89\beta - 4\beta^2)^2(1279 + 132\beta - 18\beta^2)/J^2$
$CS(g^{p_0}) = 2(a-\bar{c})^2(19855 + 7277\beta - 5441\beta^2 + 104\beta^3 + 6\beta^4)^2/J^2$
$W(g^{p_0}) = (a-\bar{c})^2M/J^2$

Table 8: Public partial network

Private isolated firm
$e(g^{p_0}) = 22(a-\bar{c})(285 + 24\beta - 122\beta^2 + 6\beta^3)/J$
$q(g^{p_0}) = 39(a-\bar{c})(285 + 24\beta - 122\beta^2 + 6\beta^3)/J$
$\pi(g^{p_0}) = 2558(a-\bar{c})^2(285 + 24\beta - 122\beta^2 + 6\beta^3)^2/J^2$

where $J = 66785 + 16834\beta - 22472\beta^2 + 778\beta^3 + 12\beta^4$, $K = 243377175 + 169536240\beta - 234233827\beta^2 - 108071680\beta^3 + 35156064\beta^4 + 553864\beta^5 - 142556\beta^6 - 624\beta^7 + 144\beta^8$ and $M = 1447366325 + 933679760\beta - 711960203\beta^2 - 266578268\beta^3 + 136328638\beta^4 - 5250000\beta^5 - 159996\beta^6 + 1872\beta^7 + 216\beta^8$.

A.6. The empty network

In the empty network, g^e, there are no collaboration ties. Therefore, firms cannot acquire part of their rivals' knowledge, given the absence of spillovers. The associated costs structures are given by $c_i(g^e) = \bar{c} - e_i + 2q_i,\ i \in \{0, 1, 2\}$. Standard calculations yield the following equilibrium outcomes

Table 9: Empty network

Public firm	
$e_0(g^e) = 83(a - \bar{c})/703$	$q_0(g^e) = 184(a - \bar{c})/703$
$\pi_0(g^e) = 26967(a - \bar{c})^2/494209$	$CS(g^e) = 242(a - \bar{c})^2/1369$
$W(g^e) = 160373(a - \bar{c})^2/494209$	
Private firms	
$e(g^e) = 66(a - \bar{c})/703$	$q(g^e) = 117(a - \bar{c})/703$
$\pi(g^e) = 23022(a - \bar{c})^2/494209$	

Symmetric networks with four firms

A.7. The network of degree $k = 3$

The cost structure under g^3 is given by $c_i(g^c) = \bar{c} - e_i - \beta(e_j + e_k + e_l) + 2q_i$, $i \neq j \neq k \neq l$, $i, j, k, l \in \{0, 1, 2, 3\}$. Equilibrium outcomes for a private firm's profits and social welfare are as follows

$$\pi_i(g^3) = 14(a - \bar{c})^2(3 + \beta)^2(241 + 52\beta - 14\beta^2)/\Delta^2,\ i \neq 0;$$

$$W(g^3) = (a - \bar{c})^2(314996 + 356062\beta - 170595\beta^2 - 159872\beta^3 + 53366\beta^4)/\Delta^2;$$

where $\Delta = 941 + 290\beta - 508\beta^2 + 42\beta^3$.

A.8. The network of degree $k = 2$

The cost structures under g^2 are as follows

$$\begin{aligned} c_0(g^2) &= \bar{c} - e_0 - \beta(e_1 + e_3) - (\beta/2)e_2 + 2q_0; \\ c_1(g^2) &= \bar{c} - e_1 - \beta(e_0 + e_2) - (\beta/2)e_3 + 2q_1; \\ c_2(g^2) &= \bar{c} - e_2 - \beta(e_1 + e_3) - (\beta/2)e_0 + 2q_2; \\ c_3(g^2) &= \bar{c} - e_3 - \beta(e_0 + e_2) - (\beta/2)e_1 + 2q_3. \end{aligned}$$

Equilibrium outcomes for each private firm's profits and social welfare are readily shown to be the following

$$\pi_1(g^2) \equiv \pi_3(g^2) = 2(a - \bar{c})^2(1687 + 312\beta - 72\beta^2)(1308 + 868\beta + 90\beta^2 - 33\beta^3)^2/R^2;$$

$$\pi_2(g^2) = 4(a - \bar{c})^2(3374 + 572\beta - 121\beta^2)(654 + 427\beta - 277\beta^2 - 18\beta^3)^2/R^2;$$

$$W(g^2) = (a - \bar{c})^2\Psi/R^2;$$

where $R = 410276 + 262200\beta - 196488\beta^2 - 46257\beta^3 + 16867\beta^4 - 990\beta^5$ and $\Psi = 929939739808 + 51092193480\beta + 2567762946\beta^2 - 25872967100\beta^3 - 3223925019\beta^4 + 4159386686\beta^5 + 171831065\beta^6 - 234775710\beta^7 + 20929131\beta^8$.

A.9. The network of degree $k = 1$

The cost structures under g^1 are

$$\begin{aligned} c_0(g^1) &= \bar{c} - e_0 - \beta e_1 + 2q_0; \\ c_1(g^1) &= \bar{c} - e_1 - \beta e_0 + 2q_1; \\ c_2(g^1) &= \bar{c} - e_2 - \beta e_3 + 2q_2; \\ c_3(g^1) &= \bar{c} - e_3 - \beta e_2 + 2q_3. \end{aligned}$$

Equilibrium outcomes for private profits and social welfare are given by

$$\pi_1(g^1) = 2(a-\bar{c})^2(1687+156\beta-18\beta^2)(327+31\beta-18\beta^2+4\beta^3)^2/T^2;$$

$$\pi_2(g^1) \equiv \pi_3(g^1) = 2(a-\bar{c})^2(1687+104\beta-8\beta^2)(327+24\beta-142\beta^2+6\beta^3)^2/T^2;$$

$$W(g^1) = (a-\bar{c})^2\Omega/T^2;$$

where $T = 102569 + 7400\beta - 37988\beta^2 + 6566\beta^3 - 956\beta^4 + 24\beta^5$ and $\Omega = 3742467476 + 1181227114\beta - 2608286183\beta^2 - 152533508\beta^3 + 513708642\beta^4 - 105836968\beta^5 + 16614536\beta^6 - 1584384\beta^7 + 96000\beta^8$.

A.10. The network of degree $k = 0$

The cost structure under g^0 is $c_i(g^0) = \bar{c} - e_i + 2q_i$, $i \in \{0,1,2,3\}$. Equilibrium outcomes for private profits and social welfare are readily shown to be

$$\begin{aligned}\pi_i(g^0) &= 30366(a-\bar{c})^2/885481,\ i \neq 0;\\ W(g^0) &= 314996(a-\bar{c})^2/885481.\end{aligned}$$

Appendix 2. Proofs

Proof of Proposition 2. Since the term $a - \bar{c}$ has no influence on the results in all proofs we normalise it to 1. We first show that the complete network g^c is pairwise stable. The stability conditions $i(b)$ and $ii(b)$ are trivially satisfied since no links can be added to the complete network. There are two cases to be considered. First, we show that the pair of private firms i and k have no incentive to delete their link (condition $i(a)$). Note that if the firms do so, the resulting network of collaboration will be the public-hub star network, g^{s_0}. To prove our claim, we have to establish the relationship $\pi(g^c) > \pi(g^{s_0})$.

Notice that the subscripts are dropped due to symmetry, i.e. $\pi_i(g^c) = \pi_k(g^c) = \pi(g^c)$. Condition $i(a)$ is thus satisfied since

$$\begin{aligned} \pi(g^c) &= 2(1279 + 220\beta - 50\beta^2)/A^2 > \\ \pi(g^{s_0}) &= 2(3+\beta)^2(1279 + 176\beta - 32\beta^2)/\Theta^2, \end{aligned}$$

where A and Θ are defined as in Tables 1 and 5.

We now turn to show that the stability condition $ii(a)$ is satisfied. This condition says that the public firm j and a private firm, say k without loss of generality, are better off by not severing their link. Notice that the resulting network when firms j and k break their collaboration tie is the private-hub star network, g^s, with firm k being a "spoke" in g^s. Then it is easily established that $W(g^c) > W(g^s)$ and $\pi(g^c) > \pi(g^s)$ since

$$W(g^c) = C/A^2 > W(g^s) = L/2D^2 \text{ and}$$

$$\begin{aligned} \pi(g^c) &= 3B/A^2 > \\ \pi(g^s) &= 4(2558 + 308\beta - 49\beta^2)(570 + 293\beta - 126\beta^2 - 10\beta^3)^2/D^2, \end{aligned}$$

where A, B, C, D and L are defined as in Tables 1, 3 and 4. Therefore, we have shown that the complete network is pairwise stable. This also proves that the star networks (public-hub star and private-hub star) are not pairwise stable.

We show next that the empty network is not pairwise stable. The stability conditions $i(a)$ and $ii(a)$ are trivially satisfied because there are no links to be deleted from the empty network. However, condition $i(b)$ is not satisfied for the empty network since the private firms have an incentive to form a link. To see this, note that

$$\pi(g^e) = 23022/494209 < \pi^l(g^p) = 18(1279 + 88\beta - 8\beta^2)/H^2,$$

where H can be found in Table 6. This suffices to establish that the empty network is not pairwise stable. Alternatively, one can show that condition $ii(b)$ is violated because the public firm and a private firm have an incentive to form a collaboration tie, i.e. $W(g^e) < W(g^{p_0})$ and $\pi(g^e) < \pi^l(g^{p_0})$.

The next step is to show that the partial networks are not pairwise stable. Notice that conditions $i(a)$ and $ii(a)$ are satisfied because no pair of firms wants to sever their link. (This follows from the proof above that the empty network is not stable.) Thus it remains to show that either condition $i(b)$ or $ii(b)$ is not fulfilled so that the partial networks are not stable. We begin to show this for the private partial network, g^p. The relevant condition here is $ii(b)$. That is, a private firm, say firm i without loss of generality, and the public firm $j = 0$ are better off by forming a collaboration tie, with firm i being a "hub" in the resulting private-hub star network, g^s (violation of condition $ii(b)$). Using the equilibrium outcomes in Tables 2, 3 and 6, we have that

$$\begin{aligned} W(g^s) &= L/2D^2 > \\ W(g^p) &= (160373 - 22140\beta + 6956\beta^2 - 720\beta^3 + 80\beta^4)/H^2 \text{ and} \end{aligned}$$

$$\begin{aligned} \pi^h(g^s) &= 2(1279 + 220\beta - 50\beta^2)(1140 + 716\beta + 102\beta^2 - 21\beta^3)^2/D > \\ \pi^l(g^p) &= 18(1279 + 88\beta - 8\beta^2)/H^2. \end{aligned}$$

Thus the private partial network is not pairwise stable.

Finally, we show that the public partial network g^{p_0} is not pairwise stable. The relevant conditions here are $i(b)$ and $ii(b)$. Thus it suffices to show that any condition is violated for the public partial network to be unstable. Considering the incentives of the non-linked private firm, say k without loss of generality, and the public firm $j = 0$ to form a connecting link, then from Tables 7, 8 and 5 we have that

$$\begin{aligned} W(g^{p_0}) &= M/J^2 < \\ W(g^{s_0}) &= (160373 + 175582\beta - 70251\beta^2 - 70072\beta^3 + 21328\beta^4)/\Theta^2 \text{ and} \end{aligned}$$

$$\begin{aligned} \pi(g^{p_0}) &= 2558(a - \bar{c})^2(285 + 24\beta - 122\beta^2 + 6\beta^3)^2/J^2 < \\ \pi(g^{s_0}) &= 2(3 + \beta)^2(1279 + 176\beta - 32\beta^2)/\Theta^2, \end{aligned}$$

with firm k being a "spoke" in the resulting public-hub star network, g^{s_0}. This constitutes a violation of condition $ii(b)$ for stability and, in turn, establishes our claim. One can show instead that condition $i(b)$ is violated, because $\pi^h(g^s) > \pi^l(g^{p_0})$ and $\pi(g^s) > \pi(g^{p_0})$. The proof is now complete. Q.E.D.

Proof of Proposition 3. First we show that the public-hub star network g^{s_0} is not Pareto efficient. All firms are better off under the complete network. That is, $W(g^c) > W(g^{s_0})$ and $\pi(g^c) > \pi(g^{s_0})$. The private-hub star network g^s is not Pareto dominated by the complete network if and only if $\beta < 0.82$. This follows by noting that $W(g^c) > W(g^s)$, $\pi(g^c) > \pi^h(g^s)$ iff $\beta > 0.82$ and $\pi(g^c) > \pi(g^s)$, where h refers to the (private) hub firm in the star network. Next, we have that $W(g^c) > W(g^p)$ and $\pi(g^c) < \pi^l(g^p)$ iff $\beta < 0.13$; hence, the private partial network is not Pareto dominated by the complete network provided that $\beta < 0.13$. We proceed to show that the public partial network g^{p_0} is not Pareto efficient. The comparisons $W(g^c) > W(g^{p_0})$, $\pi(g^c) < \pi^l(g^{p_0})$ iff $\beta < 0.04$ and $\pi(g^c) > \pi(g^{p_0})$ imply that the public partial network is not Pareto dominated by the complete network whenever $\beta < 0.04$, and vice versa. However, the public partial network g^{p_0} is Pareto dominated by the private-hub star network g^s and thus it is not Pareto efficient. This follows from $W(g^s) > W(g^{p_0})$, $\pi^h(g^s) > \pi^l(g^{p_0})$ and $\pi(g^s) > \pi(g^{p_0})$. Further note that the empty network is not Pareto efficient since $W(g^c) > W(g^e)$ and $\pi(g^c) > \pi(g^e)$. The comparisons above also imply that the complete network is Pareto efficient because it is not Pareto dominated by any other network. Finally, we turn to compare the private-hub star network g^s and the private partial network g^p for $\beta \in [0, 0.13]$. We have that $W(g^s) > W(g^p)$, $\pi^h(g^s) > \pi^l(g^p)$ and $\pi(g^s) < \pi^l(g^p)$ iff $\beta < 0.92$. Hence, neither of the two networks is Pareto dominant whenever $\beta \in [0, 0.13]$. Because the private-hub star network g^s and the private partial network g^p are not Pareto dominated by the complete network when $\beta \in [0, 0.82]$ and $\beta \in [0, 0.13]$, respectively, it follows that they are Pareto efficient within each respective range of values of the spillover parameter.

Proof of Proposition 4. First we show that the complete network of degree $k = 3$, denoted by g^3, is always stable. Consider the coalition of all firms $S = 0, 1, 2, 3$. Within the context of symmetric networks g^k, with $k = 0, 1, 2, 3$, there are three possible deviations from g^3 by S (see Figure 5): (i) both pairs of the firms {0,2} and {1,3} deleting their links to form the network g^2; (ii) all pairs of the firms {0,2}, {1,3}, $\{0, 3\}$, $\{1, 2\}$ deleting their links to form the network g^1, and (iii) the coalition of all firms deleting their links to form the empty network g^0. It can be readily shown that both of the pairs {0,2} and {1,3} have an incentive to maintain their links since $W(k = 3) > W(k = 2)$ and $\pi_2(k = 3) > \pi_2(k = 2)$; $\pi_1(k = 3) > \pi_1(k = 2)$ and $\pi_3(k = 3) > \pi_3(k = 2)$. Next, we show that the network g^1 is not stable. This follows by noting that $W(k = 3) > W(k = 1)$, $\pi_1(k = 3) > \pi_1(k = 1)$, $\pi_2(k = 3) > \pi_2(k = 1)$ and $\pi_3(k = 3) > \pi_3(k = 1)$. Finally, we show that g^0 is not stable. We that $W(k = 3) > W(k = 0)$, $\pi_1(k = 3) > \pi_1(k = 0)$, $\pi_2(k = 3) > \pi_2(k = 0)$ and $\pi_3(k = 3) > \pi_3(k = 0)$. Therefore, the complete network

g^3 survives all possible deviations by a coalitions of firms within the class of symmetric networks g^k, with $k = 0, 1, 2, 3$. Consequently, g^3 is the unique (strongly) stable network. This also proves that the complete network g^3 is the unique Pareto efficient network since it Pareto dominates all other networks. Q.E.D.